SE 422
Advanced Photogrammetry

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Last week

- Similarity Transformation
  - Linear model

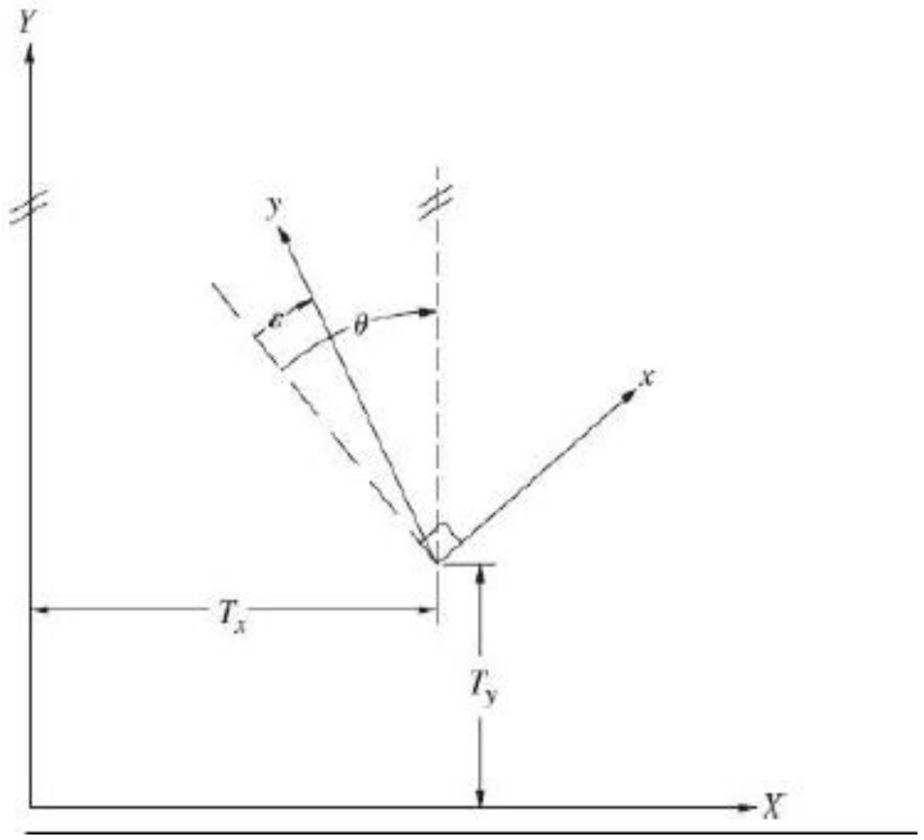
What are the differences?

  - Non-linear model
- This week, we will cover:
  - the 2D Affine transformation (linear and non-linear model)
  - The 8-parameter projective transformation (linear model only)
  - In the lab., we will practice using the MATLAB to compute and estimate the parameters of the similarity transformation (linear and nonlinear models)

2D Affine Transformation

2D Affine Transformation (6-parameter)

- The main difference between this transformation and the Similarity transformation are:
  - different scale factors in the x and y directions)
  - Compensate for nonorthogonality (non-perpendicularity) of the axis system
- This will bring the unknown parameters for a total of six

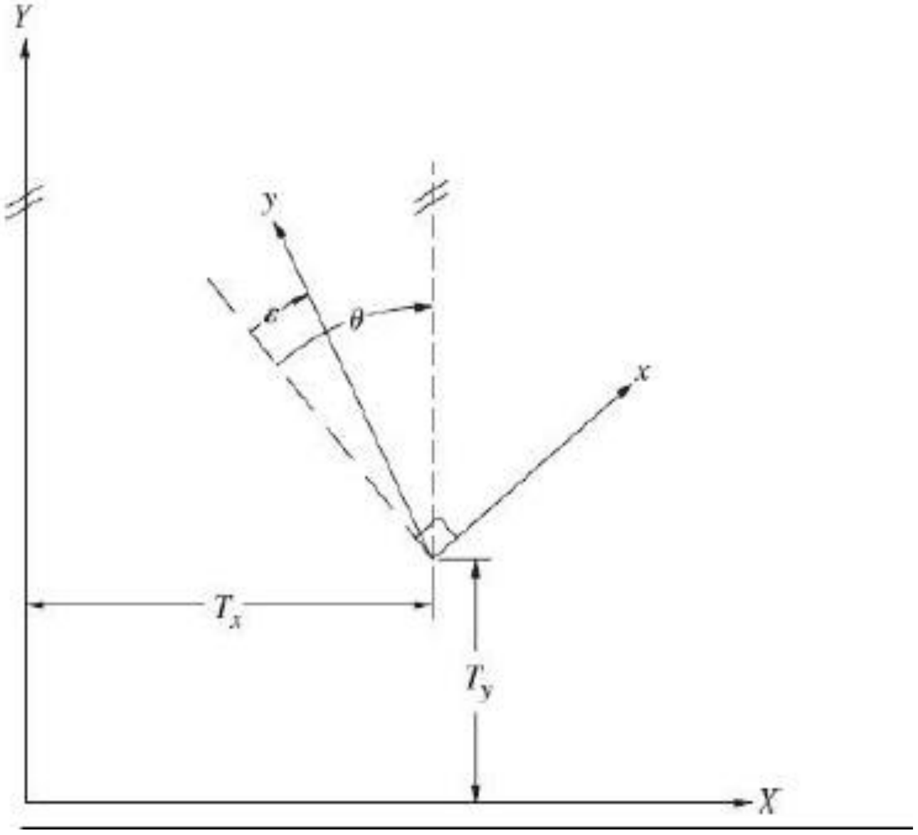


Basic steps of 2D Affine Transformations

- Scale change in x and y:

$$x' = s_x x$$
$$y' = s_y y$$

  - To make the scale of the arbitrary system (xy) equal to that of the final system (XY), each coordinate is multiplied by its associated scale factor.



Basic steps of 2D Affine Transformations

- Correction for Nonorthogonality (δ):

$$x'' = x'$$

$$y'' = \left(\frac{y'}{\cos\delta}\right) - x'\tan\delta$$

For (b)

$$x'' = x' + y'\tan\delta$$

$$y'' = y'$$

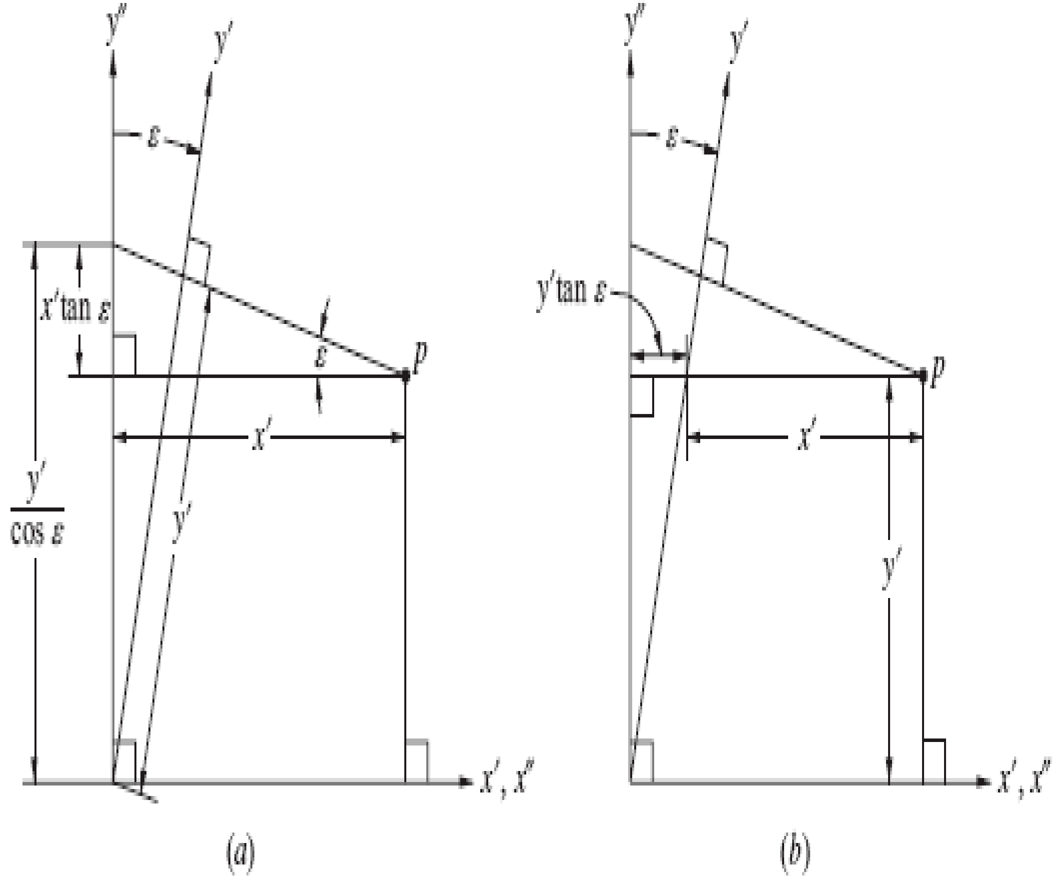


Figure C-5 (a) Two-dimensional affine relationship for typical comparator. (b) Two-dimensional affine relationship for typical scanning-type satellite image.

Courtesy of P.Wolf, B.Dweitt and B.Wilkinson

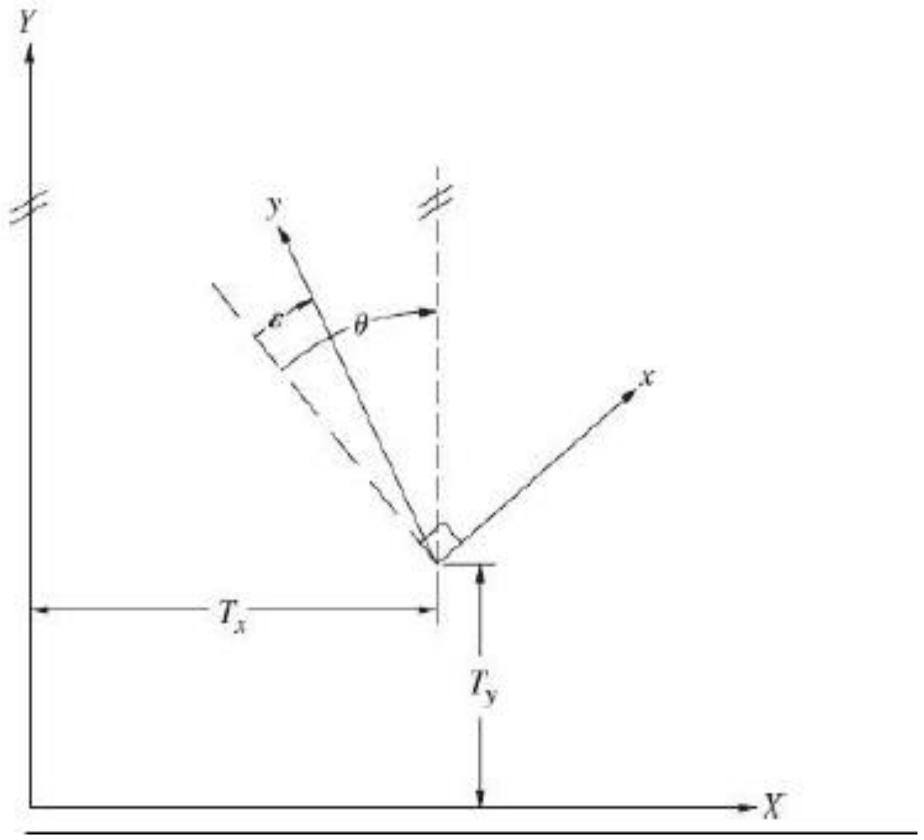
Basic steps of 2D Affine Transformations

- Rotation

$$x = x' \cos\theta - y' \sin\theta$$
$$y = y' \sin\theta + y' \cos\theta$$

- Translations

$$x = x' + tx$$
$$y = y' + ty$$



Deriving two-Dimensional (2D) Affine Transf.

- Combining all the previous four steps:

$$X = s_x\, x \cos\theta - \left(\left(\frac{s_y y}{\cos\delta}\right) - s_x\, x \tan\delta\right) \sin\theta + tx$$

$$Y = s_x\, x \sin\theta + \left(\left(\frac{s_y y}{\cos\delta}\right) - s_x\, x \tan\delta\right) \cos\theta + ty$$

- This equation can be simplified as:

$$X = s_x\, x \left(\frac{\cos(\delta - \theta)}{\cos\delta}\right) - s_y y \left(\frac{\sin\theta}{\cos\delta}\right) + tx$$

$$Y = -s_x\, x \left(\frac{\sin(\delta - \theta)}{\cos\delta}\right) + s_y y \left(\frac{\cos\theta}{\cos\delta}\right) + ty$$

Deriving two-Dimensional (2D) Affine Transf.

- We can substitute:

$a_0 = tx$

$a_1 = s_x \left(\frac{\cos(\delta - \theta)}{\cos \delta}\right)$

$a_2 = -s_y \left(\frac{\sin \theta}{\cos \delta}\right)$

$b_0 = ty$

$b_1 = -s_x \left(\frac{\sin(\delta - \theta)}{\cos \delta}\right)$

$b_2 = s_y \left(\frac{\cos \theta}{\cos \delta}\right)$

- Then,

$$X = a_0 + a_1 x + a_2 y$$
$$Y = b_0 + b_1 x + b_2 y$$

Two-Dimensional (2D) Affine Transf.

- Extracting the physical parameters:

$\boldsymbol{\theta}$	$\boldsymbol{\delta}$	$\boldsymbol{S_x}$	$\boldsymbol{S_y}$	$\boldsymbol{t_x}$	$\boldsymbol{t_y}$
$\theta = \tan^{-1}\left(-\frac{a_2}{b_2}\right)$	$\delta - \theta = \tan^{-1}\left(\frac{-b_1}{a_1}\right)$ $\delta = (\delta - \theta) + \theta$	$S_x = a_1\left(\frac{\cos\delta}{\cos(\delta - \theta)}\right)$	$S_y = b_2\left(\frac{\cos\delta}{\cos\theta}\right)$	$t_x = a_o$	$t_y = b_o$

Two-Dimensional (2D) Affine Transformation

$$X_i = a_1 x_i + a_2 y_i + a_o$$

$$Y_i = b_1 x_i + b_2 y_i + b_o$$

Linear Model

The linear model of 2D affine transformation can be used to solve for the unknowns then the parameters can be used as approximation values for the nonlinear model.

$$\begin{bmatrix} X_1 \\ . \\ . \\ . \\ Y_n \end{bmatrix}_{2n} = \begin{bmatrix} 1 & x_1 & y_1 & 0 & 0 & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 1 & x_n & y_n \end{bmatrix}_{2n\times 6} \begin{bmatrix} a_o \\ a_1 \\ a_2 \\ b_o \\ b_1 \\ b_2 \end{bmatrix}_{6x1}$$

Example:

Calibrated coordinates and comparator-measured coordinates of the four fiducial marks for a certain photograph are given in the following table. The comparator-measured coordinates of other points 1, 2, and 3 are also given. It is required to compute the corrected coordinates of points 1, 2, and 3 by using the affine transformation.

Point	(Comparator Coordinates) *x*, mm	(Comparator Coordinates) *y*, mm	(Calibrated Coordinates) *X*, mm	(Calibrated Coordinates) *Y*, mm
Fiducial *A*	228.170	129.730	112.995	0.034
Fiducial *B*	2.100	129.520	-113.006	0.005
Fiducial *C*	115.005	242.625	0.003	112.993
Fiducial *D*	115.274	16.574	-0.012	-113.000
1	206.674	123.794		
2	198.365	132.856		
3	91.505	18.956		

Solution:

- Here we have 4 fiducial points
- Putting it in LS form: $A_{8x6}X_{6x1} = L_{8x1}$
- Equations:

$$\boldsymbol{X_i} = \boldsymbol{a_1 x_i} + \boldsymbol{a_2 y_i} + \boldsymbol{a_o}$$

$$\boldsymbol{Y_i} = \boldsymbol{b_1 x_i} + \boldsymbol{b_2 y_i} + \boldsymbol{b_o}$$

$$A = \begin{bmatrix} 1 & x_a & y_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_a & y_a \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_d & x_d & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_d & x_d \end{bmatrix}, L = \begin{bmatrix} X_A \\ Y_A \\ X_B \\ Y_B \\ \vdots \\ X_D \\ Y_D \end{bmatrix}$$

Solution:

- Solving for X: $X = \left(A^T A\right)^{-1}(A^T L)$

$$X = \begin{bmatrix} \boldsymbol{a_o} \\ \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{b_o} \\ \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} = \begin{bmatrix} -115.270 \\ 0.999694 \\ 0.001256 \\ -129.479 \\ -0.000800 \\ 0.999742 \end{bmatrix}$$

$$v = AX - L$$

Solution:

- **Compute the coordinates for points 1, 2, and 3:**
- **Use:**

$$X_i = a_1 x_i + a_2 y_i + a_o$$

$$Y_i = b_1 x_i + b_2 y_i + b_o$$

Point	X (mm)	Y(mm)
1	91.496	-5.882
2	83.201	3.184
3	-23.769	-110.601

Solution:

- **Compute the physical parameters:**

$\boldsymbol{\theta}$	$\boldsymbol{\delta}$	$\boldsymbol{S_x}$	$\boldsymbol{S_y}$	$\boldsymbol{t_x}$	$\boldsymbol{t_y}$
$\theta = \tan^{-1}\left(\frac{-a_2}{b_2}\right)$ $= -0.001256$	$\delta - \theta = \tan^{-1}\left(\frac{-b_1}{a_1}\right)$ $= 0.00080$ $\delta = (\delta - \theta) + \theta$ $= -0.000456$	$S_x = a_1\left(\frac{\cos\delta}{\cos(\delta - \theta)}\right)$ $= 0.999694$	$S_y = b_2\left(\frac{\cos\delta}{\cos\theta}\right)$ $= 0.999743$	$t_x = -115.270$	$t_y = -129.479$

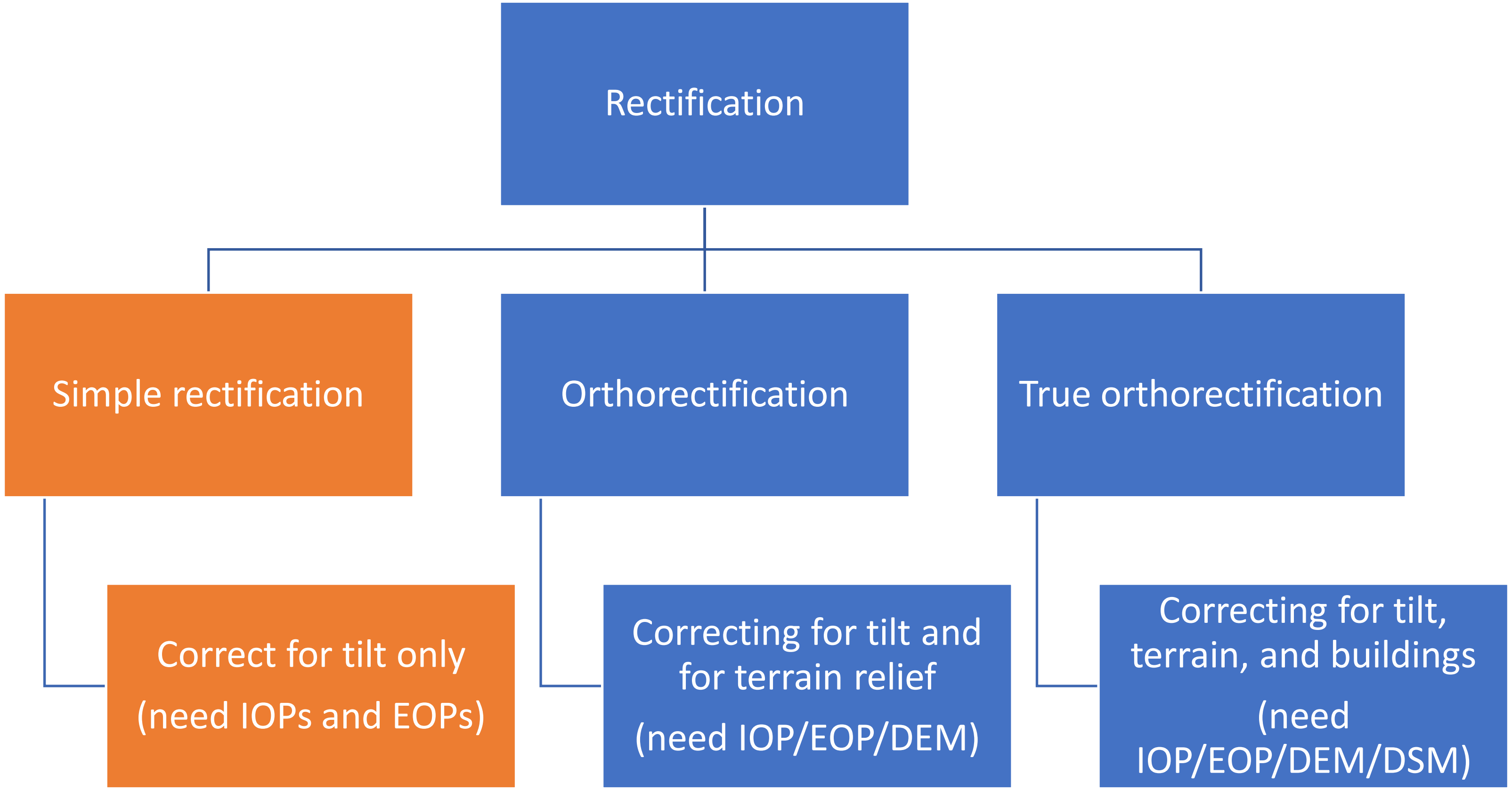
8-parameter Transformation

- The 8-parameter transformation is used to relate corresponding points in two planes that are related by perspective projection (i.e., a film plane and an object plane)

- Useful to obtain the initial approximated values of the EOPs at the time of exposure.

- Also useful for rectifying the imagery of planar objects or surfaces (by using this transformation between the image plane and the projective plane to rectify the photos)

Types of the Rectification



8-parameter Transformation (Linear Model)

- Linear Model of this projective transformation can be written as:

$$x = \frac{a_0 + a_1X + a_2Y}{1 + C_1X + C_2Y}$$

$$y = \frac{b_0 + b_1X + b_2Y}{1 + C_1X + C_2Y}$$

Parameters: $a_0, a_1, a_2, b_0, b_1, b_2,$ C_1 and C_2

$$x + xC_1X + xC_2Y = a_0 + a_1X + a_2Y$$

$$y + yC_1X + yC_2Y = b_0 + b_1X + b_2Y$$

8-parameter Transformation

- In Matrix form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & X & Y & 0 & 0 & 0 & -xX & -xY \\ 0 & 0 & 0 & 1 & X & Y & -yX & -yY \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ C_1 \\ C_2 \end{bmatrix}$$

- Each points will generate 2-equations ➔ minimum number of points for this model are 4-point.

Examples: in lab#3, I will give an example about 2D affine, and how to build the LS model

- **Next week:**
- The midterm exam will be held next week on Tuesday (10/1/2023) from 1:30 pm to 3:00 pm at ROOM 1A13.
  - HW-2 will be due on Tuesday (17/1/2023 at 11:59 pm).

- After the MIDTERM, we will start covering the 3D model transformation and the mathematical concept of photogrammetry.